Combining cheap massive commercial data and unbiased scientific survey: a zero inflated model under preferential sampling

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Motivations

Reliable stock assessment require reliable relative abundance indices:

- Unbiased or with constant multiplicative bias,
- As precise as possible (low variability)
- Acceptable for stakeholders

Major data sources:

- Scientific survey data,
- Commercial fisheries data.
Scientific Survey data

Figure: Queen Charlotte area - DFO (GroundFish division) - Focus on Dover Sole
Scientific Survey data

- Scientific campaigns are organized regularly to monitor the species of interest.
- Mostly random sampling or stratified random sampling design.
- Produce unbiased but highly variable and expensive abundance indices series.
- Stakeholders have difficulty to accept random sampling: “why sample some zone where there is no fish”? 
Scientific Survey data

Figure: QCSd - The records are the weights of Dover Sole caught.
Commercial Fisheries data

- Cheap and massive data.
- Roughly used, produce biased abundance indices.
- Stakeholders are part of the collection process.
Figure: Com. Fish.: The records are the weight of Dover Sole Catch
Two data sources

<table>
<thead>
<tr>
<th>FISHING.ID</th>
<th>YEAR</th>
<th>LAT</th>
<th>LONG</th>
<th>SWEPT AREA</th>
<th>DOVER SOLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700</td>
<td>1999</td>
<td>51.43333</td>
<td>-129.2117</td>
<td>0.770</td>
<td>0.0</td>
</tr>
<tr>
<td>6550</td>
<td>2005</td>
<td>51.06167</td>
<td>-128.2867</td>
<td>0.610</td>
<td>210.1</td>
</tr>
</tbody>
</table>
Zero-Inflated data

Continuous Zero Inflated data

- Classically, high proportion of zeros,
- Apart from 0, continuous biomass data
Data are spatially correlated

- The biomass repartition is somehow continuous,
- Data are spatially correlated,
- This correlation should be accounted for.
Location of Commercial Fisheries catch

- Fishermen target specific species,
- Location of the catch are highly related to the amount of local biomass,
- This information must be accounted for.
Modelling challenges

The resulting model should represent,

- Zero inflated,
- Spatially correlated,
- and preferentially sampled,

data,
for building a relative abundance index.
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Taking benefits of hierarchical modelling
Biomass Model: Log Gaussian Cox Process

Let $\mu(s)$ be the local abundance and define the intensity of an inhomogenous Poisson Process which may be thought as the fish repartition.
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Let $\mu(s)$ be the local abundance and define the intensity of an inhomogeneous Poisson Process which may be thought as the fish repartition.

To account for heterogeneity,

$$\log(\mu(s)) = \alpha_0 + Z(s),$$

where $Z(s)$ is a gaussian random field (GRF) with covariance function

$$c(s, t) = \exp - \frac{d(s, t)^2}{2\phi^2}.$$
Observation Process: Compound Poisson Process

One fishing event, for a given swept area $A$:

$$N(A) \sim \mathcal{P} \left( \int_A \mu(s) ds \right),$$

is the number of fish caught.
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Approximation:

$$\int_A \mu(s) ds \approx |A| \mu s_A.$$

$$Y(A) = \sum_{i=1}^{N(A)} \xi_i,$$

where $\xi_i$ are iid random variable (weight).
Observation Process: Compound Poisson Process

**Commercial**

\[ Y^C_s = \sum_{i=1}^{N^C_s} \xi_{s,i}, \]
\[ N^C_s \sim \text{Poisson}(|A_s|\mu_s), \]
\[ \xi^C_{s,i} \sim \text{Exp}(\rho), \]

**Scientific**

\[ Y^S_s = \sum_{i=1}^{N^S_s} \xi_{s,i}, \]
\[ N^S_s \sim \text{Poisson}(|A_s|\mu_s), \]
\[ \xi^S_{s,i} \sim \text{Exp}(\rho^S), \quad \rho^S = q\rho \]

\[ \mathbb{E}(Y^C_s) = \frac{\mu_s}{\rho} \]
\[ \mathbb{E}(Y^S_s) = \frac{q\mu_s}{\rho} \]

\[ \mathbb{P}(Y^i_s = 0) = \exp(-|A_s|\mu(s)) \]

called LOL model in Ancelet & al, 2010; Lecomte & al 2013
Full model specification

**Process model:**

\[ \log(\mu(s)) = \alpha_0 + Z(s), \]

where \( Z(s) \) GRF with covariance function \( c(s, t) = \exp \left( - \frac{d(s, t)^2}{2\phi^2} \right) \).

**Data model:**

\[ Y_s^k \sim LOL(\mu_s, \rho^k) \]

But dimension issues when the number of observations increase. Reduction dimension using random basis function.
A 2 D discrete convolution of a gridded (latent) structure

The points of the grid are denoted \( g = 1..G \).

\[
X(g) \sim \text{iid} \ N(0, \sigma_x^2)
\]

Convolution kernel \( K_\theta \) between any data point \( s \) and grid location \( g \)

\[
K_\theta(s, g) = \exp \left( -\frac{d^2(s, g)}{\phi^2} \right)
\]

Discrete convolution for site \( s, s = 1..S \):

\[
Z(s) = \sum_{g=1}^{G} K_\theta(s, g) X(g) + m(s)
\]

\[
m(s) = \alpha_0 + \alpha_1 \times \text{Depth}(s) + \ldots
\]
Commercial fisheries focus on area with high abundance. The position of the commercial catch are modeled as an inhomogenous Poisson point process conditionned to have $N_{Com}$ points.

$$(S_1^C, \ldots, S_{N_{Com}}^C) \sim IPP(\mu(s))$$
Full model specification with graphics
Full model specification with graphics
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Model Summary

Data:
\[ S^C = (S^C_1, \ldots, S^C_{n_{Com}}) \text{, Commercial locations of catch : Poisson Process} \]
\[ Y^C = (Y^C_1, \ldots, Y^C_{n_{Com}}) \text{, Actual commercial catch : LOL model} \]
\[ Y^S = (Y^S_1, \ldots, Y^S_{n_{Scien}}) \text{, Actual survey catch: LOL model} \]

Latent layer:
\[ Z = K_\phi X \text{, with } X = (X_1, \ldots, X_G) \text{ Indepedant centered gaussian variables, with variance } \sigma^2. \]

Parameters:
\[ \theta = (\sigma^2, \phi, \alpha, \rho^C, \rho^S) \]

Indice:
\[ I = \int_s \mu(s) ds \]
Problems - Likelihood

Complete likelihood

\[ [Y^C, Y^S, S^C, X|\theta] = [Y^C|S^C, X, \theta] [Y^S|S^S, X, \theta] [S^C|X, \theta][X|\theta]\]

Likelihood

\[ [Y^C, Y^S, S^C|\theta] = \int_X [Y^C, Y^S, S^C, X|\theta]dX \]
Computing the likelihood

\[
Y_C, Y_S, S_C | \theta \approx \frac{1}{M} \sum_{m=1}^{M} \left[ Y_C, Y_S, S_C | X_m, \theta \right], X_m \sim \left[ X | \theta \right]
\]
Computing the likelihood

Monte Carlo approximation

\[
MC : [Y^C, Y^S, S^C | \theta] \approx \frac{1}{M} \sum_{m=1}^{M} \left[ Y^C, Y^S, S^C | X^m, \theta \right], \quad X^m \sim [X | \theta]
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\]

Importance sampling approximation

\[
IS : [Y^C, Y^S, S^C | \theta] \approx \frac{1}{M} \sum_{m=1}^{M} \frac{[Y^C, Y^S, S^C | X^m, \theta][X^m | \theta]}{q_\theta(X^m)}, \quad X^m \sim q_\theta(.)
\]
Inference on parameters

Numerical optimisation of the likelihood.

Full Metropolis Hasting algorithm.

Pseudo Marginalized MCMC Algorithm - Andrieu & Roberts (2009)

\[ \theta^* \sim q(\cdot|\theta) \text{ and } X^* \sim \text{iid } M \prod_{m=1}^{M} q_S(Z(m)|\theta^*) \]

accept them with probability

\[ \rho((X,\theta), (X^*,\theta^*)) = \tilde{\pi}(\theta^*,Z^*) \cdot \tilde{\pi}(\theta,Z) \times q(\theta^*|\theta) \cdot q(\theta|\theta^*) \]

with \( \tilde{\pi}(\theta,Z) \) given by the importance sampling quantity

\[ \tilde{\pi}(\theta,Z) = \frac{1}{M} \sum_{m=1}^{M} [Y_C, Y_S, S_C, Z(m),\theta] \cdot q_S(Z(m)|\theta) \]
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Finding good importance function $q_\theta$

Moment based method with kernel smoothing:

$E(Y_s) = \mu(s) \rho_s, \hat{Z}(s) = \log(\rho\hat{Y}_s A_s), \hat{Y}(s) = K_{\text{smooth}} Y$,

$P(Y_s = 0) = \exp\{-|A_s| \mu(s)\}$,

$\tilde{Z}_s = \log\left(-\log(\tilde{p}_s)/|A_s|\right), \tilde{p}_s = \#\text{Neighbours with 0}$

Finally, $Z = KX, \hat{X} = (K'K)^{-1}K'\left(p\tilde{Z} + (1-p)\hat{Z}\right)$.

$X \sim N(\hat{X}, \Sigma_X)$.
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And now

Mixing all the ingredients and baking the cake


