# Influence of Sites and Species on Phylogenetic Stability

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### Main Goal:

Use biological macromolecules (DNA, proteins) to unravel the evolutionary history of a set of species

### **Basic Ideas:**

- Closely related species: highly similar molecules,
- Distantly related species: not so similar molecules,
- Use similarity information to reconstruct probable evolution,

#### **Results:**

- Evolution is assumed to be tree-like,
- Results are displayed as a phylogenetic tree.

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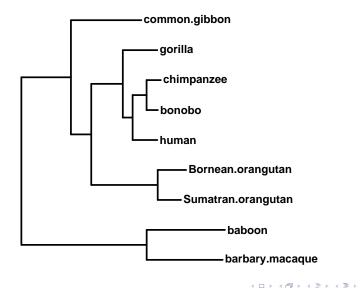
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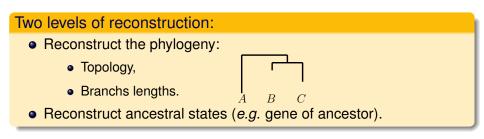
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# Example of A Phylogenetic Tree





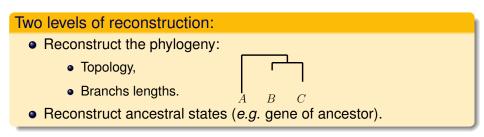
# **Reconstructions and Limits**



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Almost no direct observations or results on the evolutionary process.

**Collection:** Select gene/protein shared by all species, sequence it and align the sequences.

## Example:

• Alignment  $\mathcal{X} = (X_{ij})$  of size  $s \times n$  (6 species  $\times$  10 sites)

Fin Whale	М	Ν	Ε	Ν	L	F	Α	Р	F
Blue Whale	М	Ν	E	Ν	L	F	Α	Р	F
Chimpanzee	Μ	Ν	Ε	Ν	L	F	Α	S	F
Bonobo	Μ	Ν	Ε	Ν	L	F	Α	S	F
Gorilla	Μ	Ν	Ε	Ν	L	F	Α	S	F
Bornean Orangutan	М	Ν	Ε	D	L	F	Т	Р	F

•  $\mathcal{X}_{24} = \mathbf{N}$ ,

• 4th site:  $\mathbf{X}_4 = (\mathbf{NNNNND})'$ ,

•  $2^{nd}$  species (Harbor Seal):  $\mathbf{X}^{(2)} = \underline{MNENLFAPFM}$ .

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All methods infer the tree which minimize/maximize a given criteria:

- Maximum Parsimony: minimizes the number of changes needed to explain the current data;
- Neighbor-Joining: minimizes a natural estimate of the tree length;
- Maximum Likelihood: maximizes the likelihood of the data;
- Bayesian: maximizes the posterior probability of the data.

# **Inference Method**

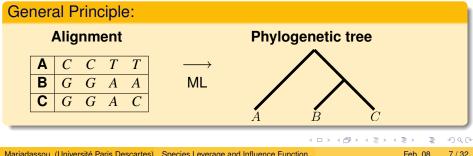
### Likelihood Based

- Assume (**X**<sub>*i*</sub>) *i.i.d.*;
- Choose generating evolution model  $M(T, \theta_T)$ ;
- Discrete topology *T* and continuous parameter model;
- Retrieve  $(\hat{T}, \hat{\theta}_{\hat{T}})$  maximizing  $\mathbb{P}((\mathbf{X}_i); M, T, \theta_T)$ .

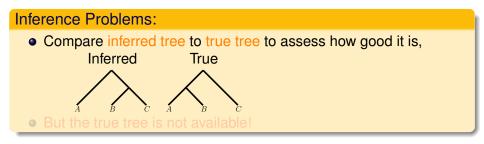
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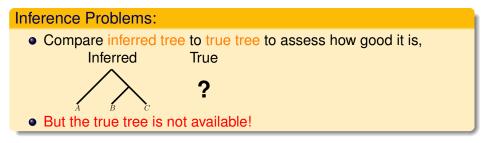
#### Confidence Issue:

- How confident are we on the inferred tree ?
- Which parts of the tree are reliable/not reliable ?

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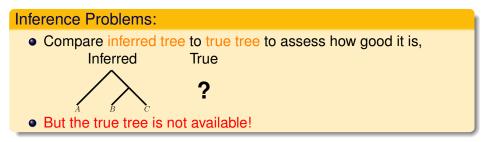
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## Confidence or Robustness ?

Confidence: Another (independent) data set gives (roughly) the same inferred tree;

Robustness: Tweaking the (original) data set gives (roughly) the same inferred tree.

### Robustness

 Most (if not all) available procedures are designed to check robustness, not confidence;

• The inferred tree might be far from the true tree, as long it is consistently so, we are happy.

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## **Bootstrap Strong Points:**

- Many potential causes for uncertainty:
  - Finite sequence lengths,
  - Poor alignment quality (outlier sites),
  - Poor species sampling (rogue species),
  - Model misspecification,
  - ...

Global measure of uncertainty,

#### Bootstrap Weak Points:

- Global measure of uncertainty,
- Unable to breakdown the uncertainty,
- Unable to pinpoint local sources of uncertainties,
- Several other ways to tweak the data.

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# **Outlier Sites: Motivation and Goal**

## Motivation: Filter Data

Sites source of errors:

- Sequencing errors;
- Alignment errors;
- Presence of an atypical DNA segment;

#### Goal

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• Quantify the influence of each site on the tree;

- Detect outlier sites;
- Infer a robust tree.

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# About the Influence Function

### Influence Function: Definition

Let  $X_1, \ldots, X_n$  be *i.i.d.* with common d.f. F on  $\mathcal{R}^d$  and S(F) a functional of F. The influence function:

$$IF_{S,F}(x) = \lim_{\varepsilon \to 0} \frac{S[(1-\varepsilon)F + \varepsilon \delta_x] - S[F]}{\varepsilon}$$

measure the influence of a perturbation in direction x.

### Empirical Version

For unknown *S* and finite size sample,  $F \to F_n = \frac{1}{n} \sum_{j=1}^n \delta_{X_j}$ ,  $\varepsilon \to -1/(n-1)$ :  $IF_{S,F_n}(X_i) = \lim_{\varepsilon \to 0} \frac{S[(1-\varepsilon)F_n + \varepsilon \delta_{X_i}] - S[F_n]}{(n-1)(S(F_n) - S(F_{n,-i}))}$ 

### where $F_{n,-i}$ is the empirical distribution on all sites but *i*.

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# And for Phylogenies...

## Definition

Let:

- $\mathbf{X} = (\mathbf{X}_1, \dots, \mathbf{X}_n)$  be the complete alignment,
- $\mathbf{X}_{-i} = \mathbf{X} \setminus \mathbf{X}_i$  all the sites but site *i*,
- $(\hat{T}, \hat{\theta}_{\hat{T}})$  the ML tree and associated parameters for X,
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- The statistic be:

$$l_{\hat{T}}(\hat{\theta}_{\hat{T}}|\mathbf{X}) = \frac{1}{n} \sum_{i=1}^{n} \log \mathbb{P}(\mathbf{X}_i | \hat{T}, \hat{\theta}_{\hat{T}})$$

The influence value of  $\mathbf{X}_i$  is then:

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# **Influence Values**

### Interpretation

- Positive value: enhanced support for the ML tree;
- Negative value: weakened support for the ML tree;
- Absolute value: strength of the support/disagreement;
- Many sites with small positive values and a few sites with large negative values.

### Strategy towards greater stability

- Focus on outliers: sites with  $IF(\mathbf{X}_i) < 0$ ;
- Rank them in increasing  $IF(\mathbf{X}_i)$ ;
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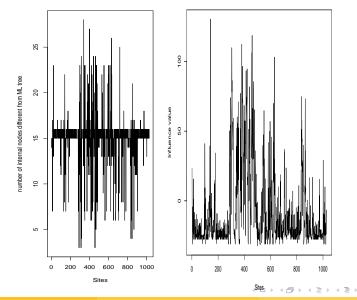
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- Lower mushrooms"
- Biology: widely unknown!
- Strong enough phylogenetic signal to correctly resolve the topology.
- 1026 sites, 158 OTUs, GTR model

# Information about sites



M. Mariadassou (Université Paris Descartes) Species Leverage and Influence Function

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## Distance between trees

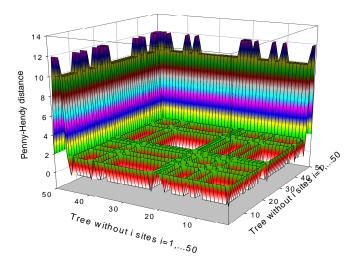
0	20	18	18	18	18	18	18	18	20
20	0	2	2	2	2	2	2	2	2
18	2	0	0	0	0	0	0	0	2
18	2	0	0	0	0	0	0	0	2
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20	2	2	2	2	2	2	2	2	0

 $T_i$ : trees constructed without the *i* most influent sites.  $D_{ii}$ : Robinson-Foulds distance between  $T_i$  and  $T_i$ 

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## **Distance Between Trees**

Distance between trees



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## **Bootstrap Strong Points:**

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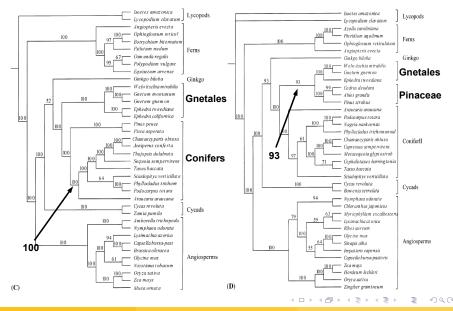
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# Seed Plant Phylogeny (Ridyn & al. 2002)



M. Mariadassou (Université Paris Descartes) Species Leverage and Influence Function



# Species Leverage Index: Motivation and Goal

## Species Leverage Index (SLI)

• Goal: Study the robustness of the tree with respect to the species,

 Motivation: Thanks to strange evolutionary features not taken into account by the inference method, some species may exert a strong pull toward a biased estimated phylogeny,

### • Method:

- Infer the phylogeny T with the whole species set,
- Remove species one at the time and infer a new tree *T<sub>i</sub>* on the smaller species set,
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### Definition

Let:

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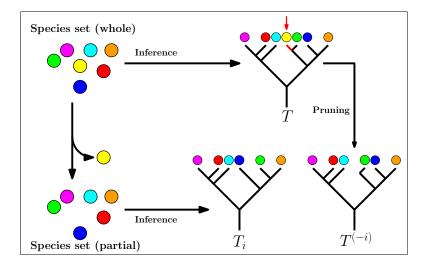
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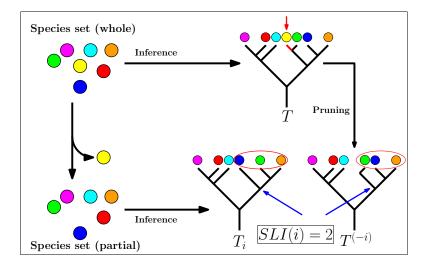




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## **Method**





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## Nodes Leverage Index (NLI)

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Node Leverage Index (NLI) of *A*: number of inferred trees in which the node is retrieved.

$$NLI(A) = \sum_{i=1}^{n} \mathbb{1}_{\widehat{T^{(-i)}}}(A)$$

#### Problems

- The taxa sets are different between  $\hat{T}$ ,  $\hat{T}^{(-i)}$  and  $\hat{T}^{(-i)}$ ;
- The taxa sets are different between the  $T^{(-i)}$ s;
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#### Interpretation

- SLI: Low value: adding/removing the species from the dataset has (almost) impact on the tree;
  - High value: "rogue" species, adding/removing it greatly affects the tree.
- NLI: High value: stable nodes, highly resilient to taxon sampling;
  - Low value: weak nodes, highly sensitive to taxon sampling.

#### Strategy towards robustness

- Focus on rogues species: species with high SLI;
- Rank them in increasing SLI;
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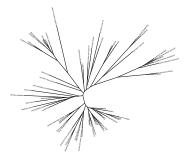
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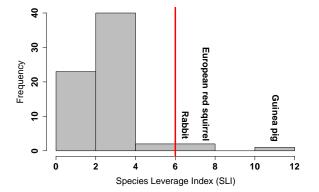
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## Data: Placental Mammal Phylogeny

- Mitochondrial genome of 68 mammals,
- Amino Acids sequences,
- Sequences are 3658 sites long,
- Phylogeny published in Nikaido et al. in 2003.



### Species Leverage Index



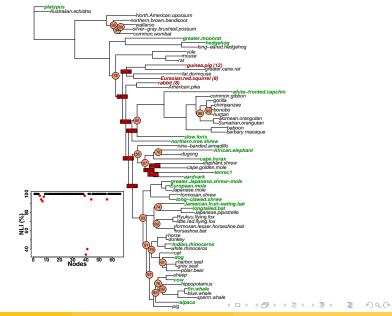


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## Complete Phylogeny



M. Mariadassou (Université Paris Descartes) Species Leverage and Influence Function

# **Rogue Species**





### Two sources of uncertainties

- Outlier sites;
- Rogue species.

#### Two tools to detect them

- Influence functions;
- Species Leverage.

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### Two sources of uncertainties

- Outlier sites;
- Rogue species.

### Two tools to detect them

- Influence functions;
- Species Leverage.

#### Conclusions

- Bootstrap: global measure of uncertainty,
- SLI,NLI are local ones to pinpoint the sources of uncertainties,
- Decompose the "black box" of bootstrap values,

#### Perspectives

- Impact of the evolution model,
- Statical properties of SLI,NLI.

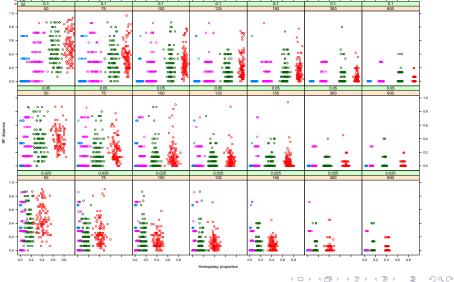
### Add-on

 Homoplasy influence on accuracy

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 02
 04
 08
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 02
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 08

 01
 0.1
 0.1
 0.1
 0.1
 0.1
 0.1

 75
 100
 125
 150



0.0 0.2 0.4 0.6 0.8

