

# Free Model for Generalized Path Modeling and Comparison with Bayesian Networks

Christian Derquenne



*Research and Development*

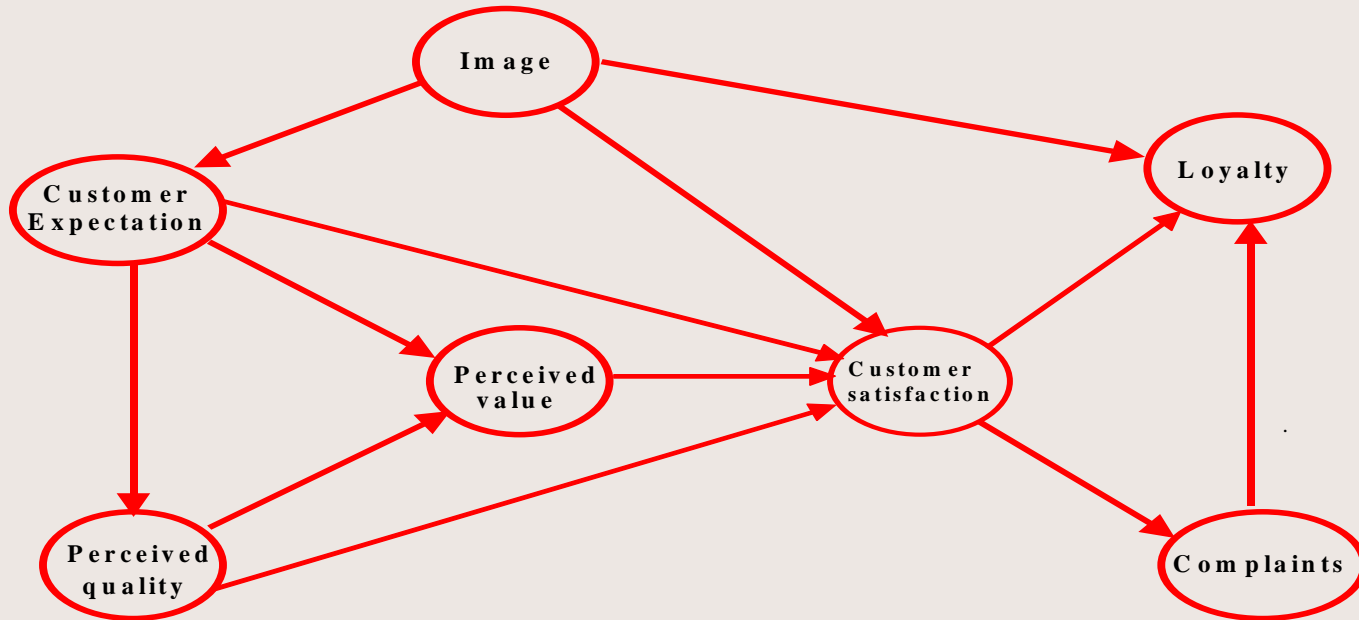


# Outline

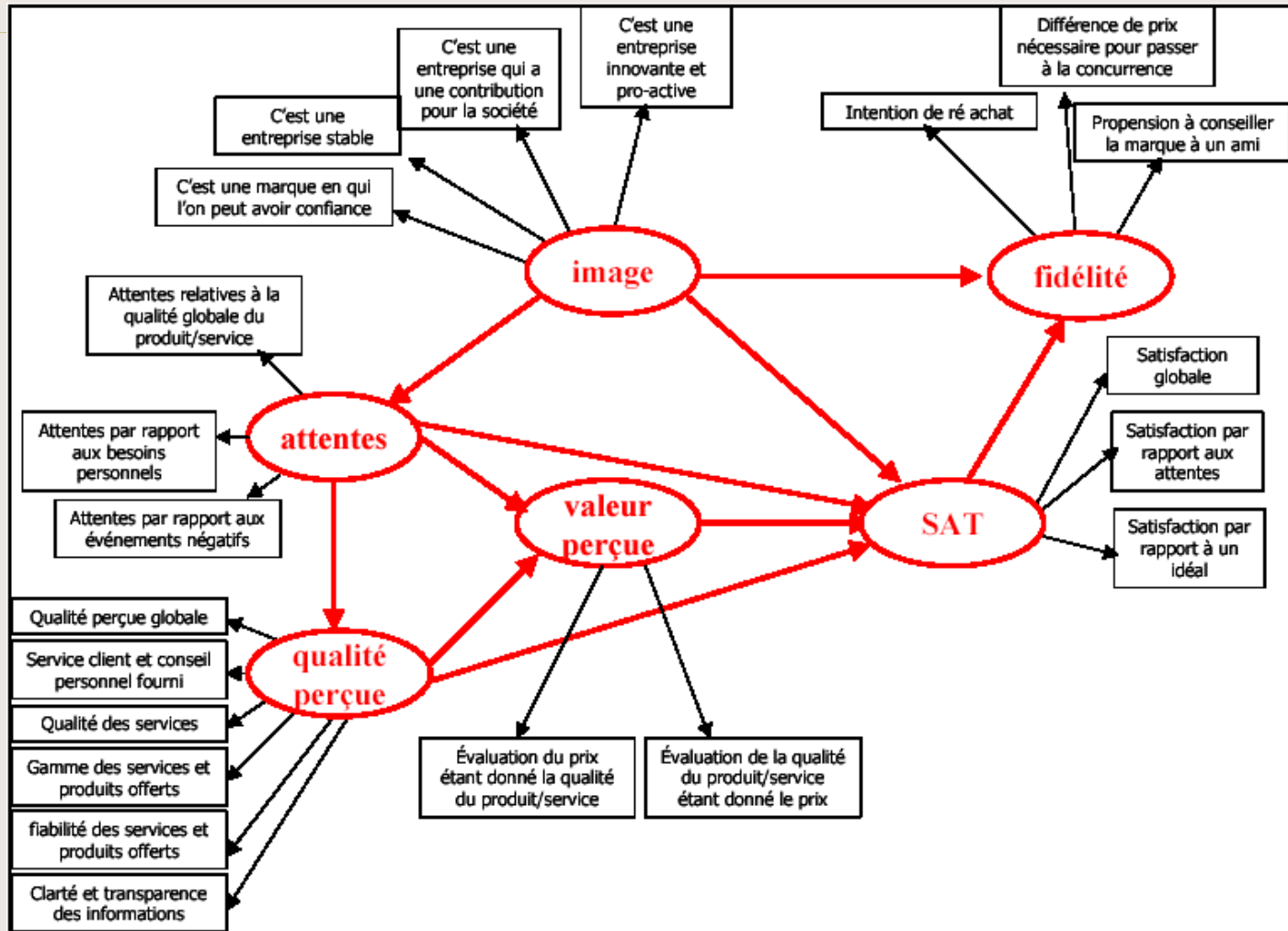
- 1 Issues and motivations
- 2 How to build a free model ?
- 3 Generalized Free Model approach
- 4 Bayesian Networks approach
- 5 Application to marketing data
- 6 Concluding remarks, applications and perspectives

# Issues and motivations

- ECSI model (or others) are defined by the experts whom are the knowledge of their application domains (marketing, policy, economics, sensiometrics...)



# Issues and motivations



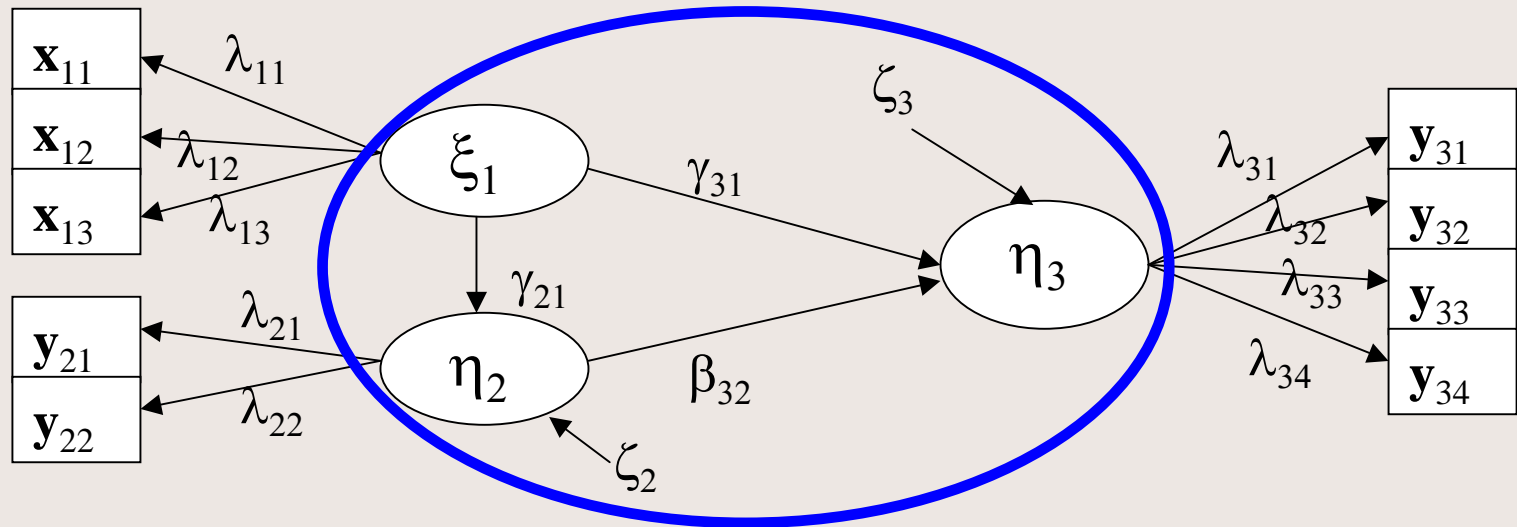
# Issues and motivations

## Un peu de vocabulaire :

- Variable latente **exogène** = {Image}
- Variables latentes **endogènes** = {Qualité perçue ; Attentes ; Valeur perçue ; Satisfaction ; Fidélité}
- Variable latente **cible** = {Fidélité}
- Variables manifestes liées au bloc « **Attentes** »  
{Attentes relatives à la qualité globale du produit/service ;  
Attentes par rapport aux besoins personnels ;  
Attentes par rapport aux événements négatifs }
- Modèle de mesure (externe) : lien entre les variables manifestes et les variables latentes
- Modèle de structure (interne) : lien entre les variables latentes

# Issues and motivations

## Structural model (internal)



$\xi_i, \eta_j$ : latent variables

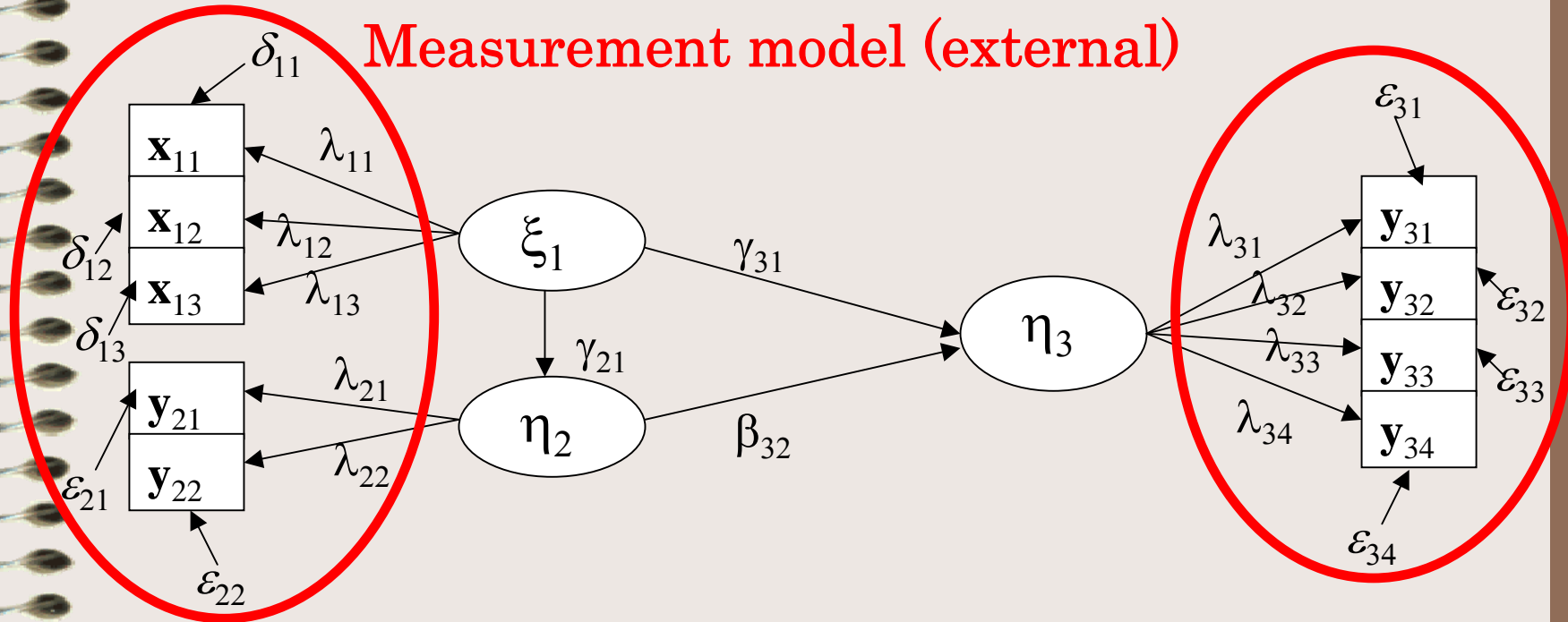
$$\eta_2 = \gamma_{21}\xi_1 + \zeta_2$$

$\beta_{23}, \gamma_{21}$  et  $\gamma_{31}$ : path coefficients

$$\eta_3 = \gamma_{31}\xi_1 + \beta_{32}\eta_2 + \zeta_3$$

# Issues and motivations

## Measurement model (external)



$\mathbf{x}_{jh}$ : manifest var. (observed) of exogenous latent variables

$$\mathbf{x}_{1h} = \lambda_{1h}\xi_1 + \delta_{1h}, h = 1, 2, 3$$

$\lambda_{1h}$ : external weight

$\mathbf{y}_{jh}$ : manifest var. (observed) of endogenous latent variables

$$\mathbf{y}_{jh} = \lambda_{jh}\eta_j + \varepsilon_{jh} \quad \forall j, h$$

$\lambda_{jh}$ : external weight

# Méthodes d'estimation de ces modèles complexes

## Deux types de structure des données

- Matrice de covariances (corrélations) entre les variables manifestes : **Méthode LISREL (LInear Structural RELationships)** : Minimise l'écart entre les matrices de covariances théorique (issues du modèle) et observée. Données Gaussiennes – EMV. [\[Jöreskog, 1979\]](#)
- Matrice individu×variables : **Approche PLS (Partial Least Squares)** [\[Wold H., 1982\]](#) et **approche RFPC (Regression on the First Principal Components)** [\[Derquenne Ch., 2001\]](#) : Minimise la variance résiduelle de toutes les variables à expliquer du modèle. Pas d'hypothèse de normalité, moindres carrés ordinaires.



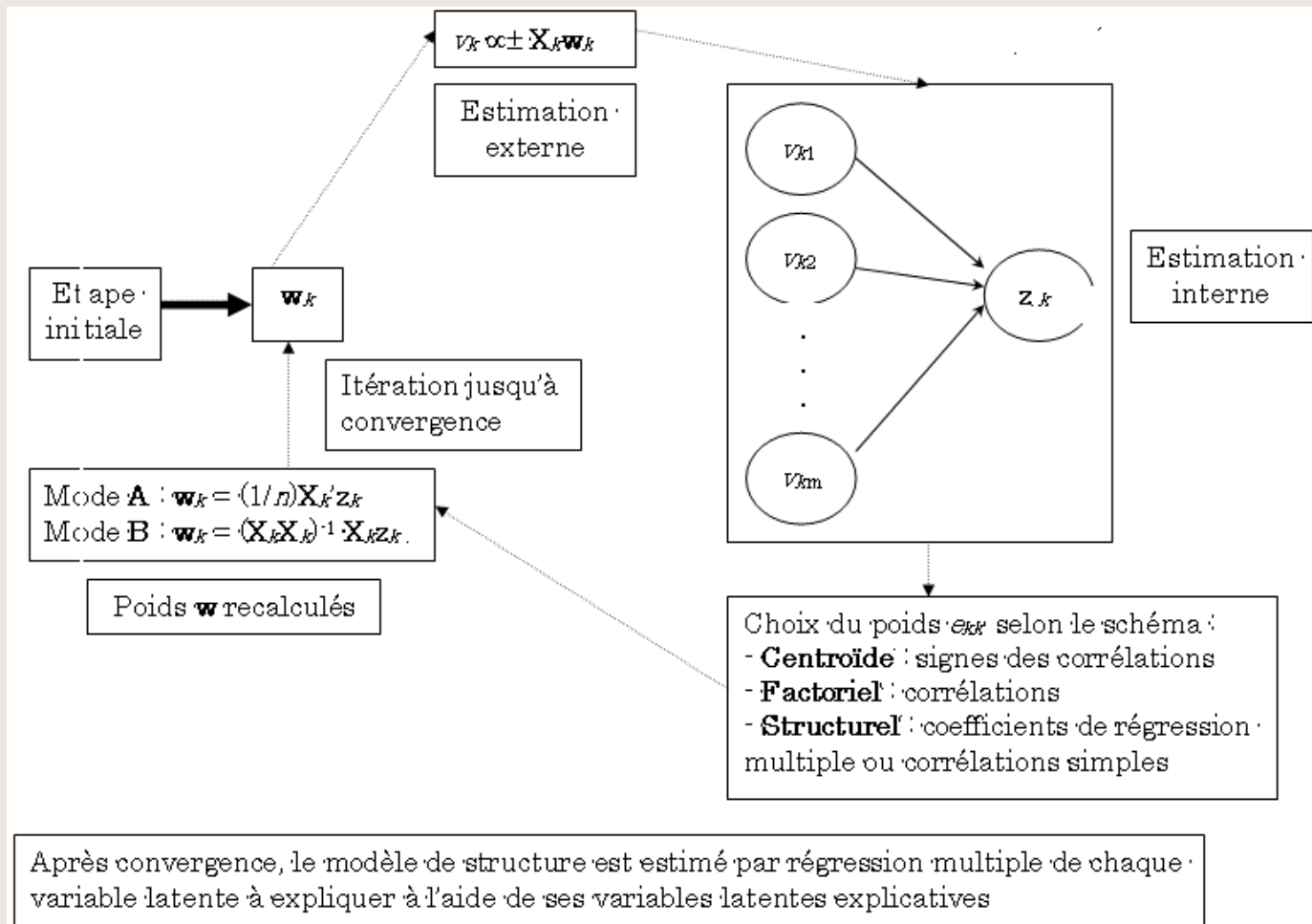
# Structure de données individuelles : l'approche PLS

## Caractéristiques principales :

- Analyse de la structure du tableau des **individus**
- **Estimations séparées** des modèles de mesure et de structure (pas de critère global à optimiser)
- Estimation par la méthode **des moindres carrés ordinaires (PLS = Partial Least Squares)**
- **Mode réflexif et formatif** pour le modèle externe
- Méthode développée par **H. Wold** (1982, 1985)

# Structure de données individuelles : l'approche PLS

## Synoptique de l'estimation du modèle



# L'approche RFPC

## Caractéristiques principales :

- Analyse de la structure du tableau des **individus**
- **Estimations séparées** des modèles de mesure et de structure (pas de critère global à optimiser)
- Utilisation de l'ACP pour estimer les variables latentes
- ⇒ Pas de système de calculs alternés entre le modèle interne et externe
- ⇒ Pas de problème de biais des paramètres
- **Mode réflexif** seulement pour le modèle externe

# Issues and motivations

- ECSI model (or others) are defined by the experts whom are the knowledge of their application domains (marketing, policy, economics, sensiometrics...)

⇒ but these models are based on strong working hypothesis

⇒ fixed models

*Based on SEM (Structural Equations Modelling):*

- Covariance (LISREL) [Jöreskog, 1979]
- PLS Approach [Wold H, 1982]
- RFPC Approach [Derquenne Ch., 2001]

# Issues and motivations

These models are fixed at four main levels :

- (i) Choice of the manifest variables corresponding to the questions of survey, for instance

Im1

Sat2

Cpt4

Im2

Sat3

Fid1

Im3

Cpt2

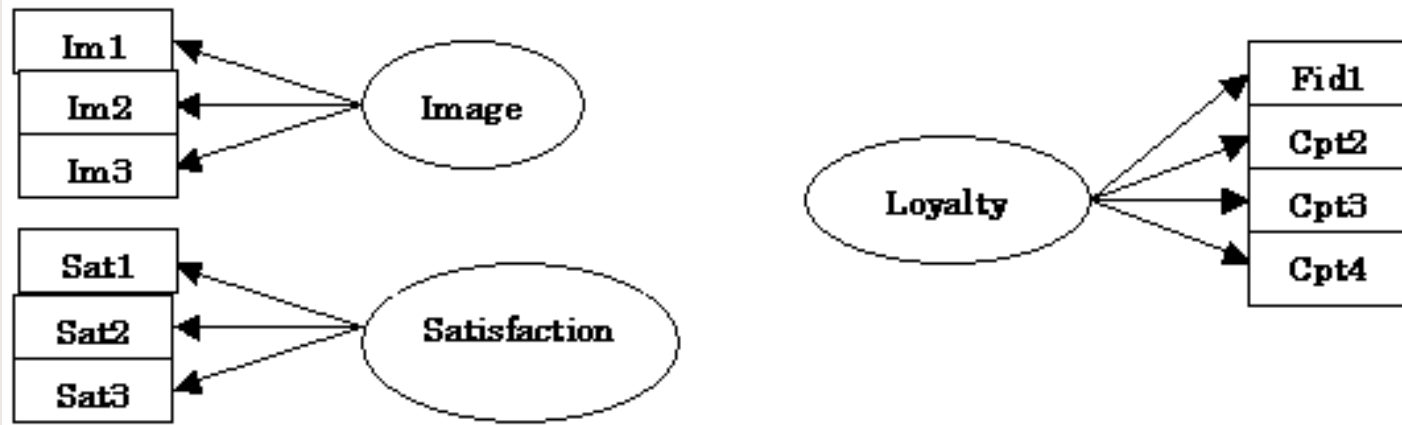
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Cpt3

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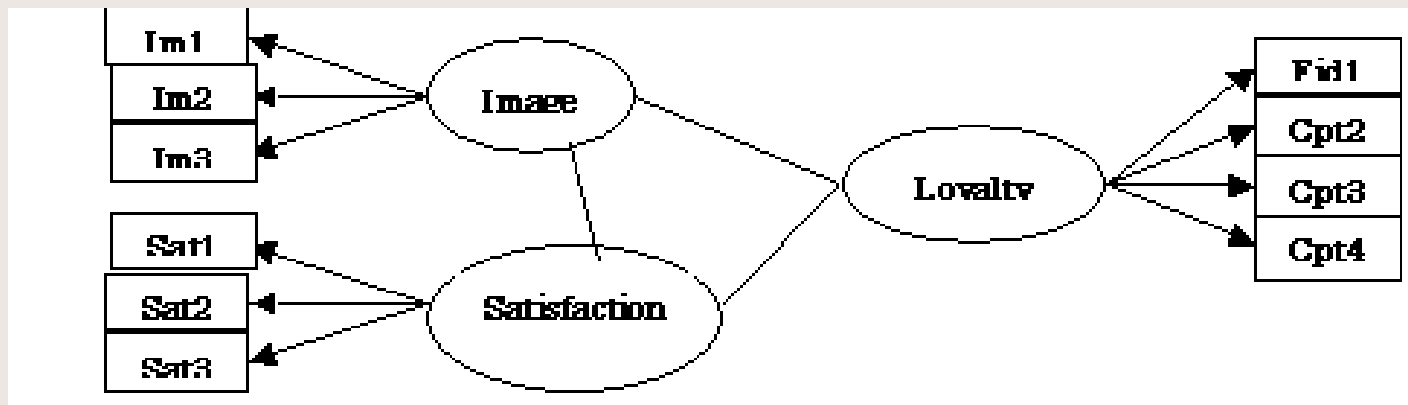
- (i) Choice of the manifest variables corresponding to the questions of survey, for instance
- (ii) Choice of blocks of manifest variables constituting the latent variables



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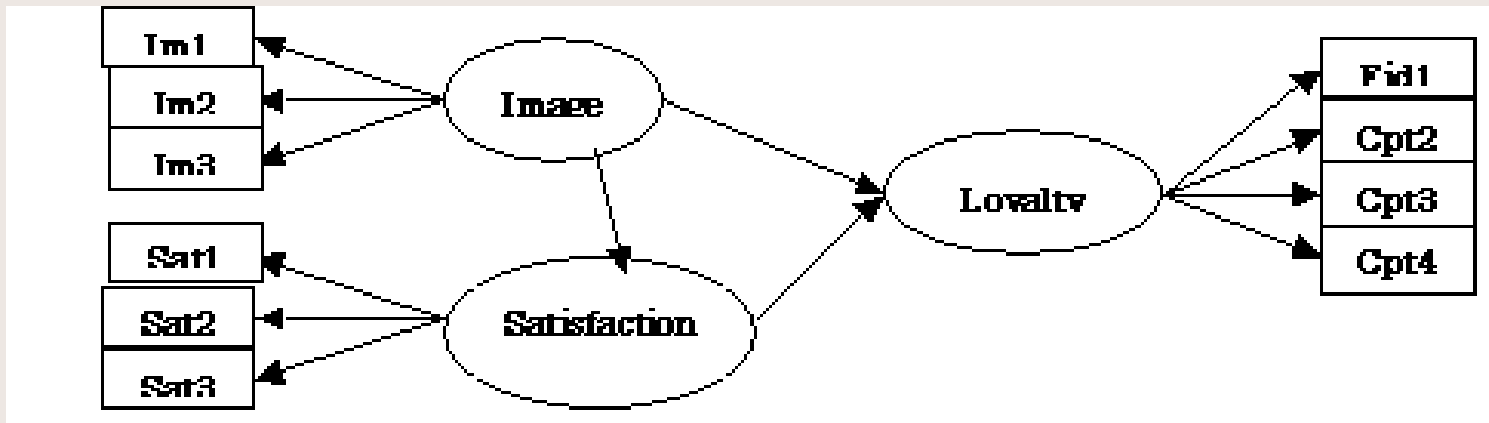
- (ii) Choice of blocks of manifest variables constituting the latent variables
- (iii) Determination of presence or absence of **links between blocks**



# Issues and motivations

These models are fixed at four main levels :

- (iii) Determination of presence or absence of links between blocks
- (iv) Orientation of the links





# Issues and motivations

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The last three levels can be thrown back into question

⇒ We can leave “to talk the data”

⇒ Using of exploratory analysis

**Feasible solution : A free model approach**

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- ⇒ We can leave “to talk the data”
- ⇒ Using of exploratory analysis

**Feasible solution : A free model approach**

- ⇒ A graphical model based on “causalities” (correlations) explored by direct/indirect observations



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# How to build a free model ?

## The free model in five steps (numeric scale)

*Step 1* : Constituting of block of manifest variables

→ Clustering of manifest variables (Factor Analysis - rotation)

*Step 2* : Building the latent variables

→ First component of Principal Components Analysis

*Step 3* : Searching for links between latent variables

→ Using of partial correlation (Pearson or Spearman)

*Step 4* : Orientation of links between latent variables

→ Usually made by expert, but possibility to use criteria: global  $R^2$ , GoF [Amato et al., 2004]

*Step 5* : Estimation of the free model

→ Regression on the First Principal Components (RFPC)  
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# How to build a free model ?

**Context** : In the applications of path modelling, data can be measured on different scales (survey questionnaires, databases, metadata, ...)

→ *pseudo-numeric* (score from 1 to 10)

→ *semantic* ({yes ; no} or {very satisfy ; nearly satisfy ; nearly not satisfy ; not satisfy}, {electricity ; gas ; fuel ; other},...)

**Problem** : Step 1 and step 2 work only with manifest variables on a numeric scale

**Solution** : To generalize the free model approach to other scales of manifest variables (binary, ordinal, nominal, ...)

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# Generalized Free Model

## The Generalized Free Model (GFM) in five steps

*Step 1*: Constituting of block of manifest variables

→ Clustering of manifest variables (Exponential Family + Generalized Linear Models)

*Step 2*: Building the latent variables

→ First component of the GPCA (Generalized NIPALS) [Derquenne, 2005]

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*Step 5*: Estimation of the generalized free model

→ Regression on the First Generalized Principal Components (RFGPC) (or PML Approach) [Derquenne, 2005]

# Generalized Free Model

## *Step 1: Clustering of manifest variables (1)*

Let  $(X_1, \dots, X_p)$  be,  $p$  manifest variables measured on different scales

*Principle*: Each variable (response variable) is modeled by the  $p - 1$  others (explanatory variables), through an adapted model

*Example*: if  $X_j$  is ordinal dependent variable with  $R_j$  categories

$\Rightarrow$  Ordinal Logit Model: Cumulated probability to category  $r_j$ :

$$\Pr[X_j \leq r_j / X_k, k = 1, \dots, p, k \neq j] = \exp\left(\alpha_{r_j} + \sum_{k \neq j} x_k \beta_k\right) / \left(1 + \exp\left(\alpha_{r_j} + \sum_{k \neq j} x_k \beta_k\right)\right) \quad (1)$$

*Example*: if  $X_j$  is nominal dependent variable with  $R_j$  categories

$\Rightarrow$  Multinomial Logit Model: Individual probability for category  $r_j$ :

$$\Pr[X_j = r_j / X_k, k = 1, \dots, p, k \neq j] = \exp\left(\alpha_{r_j} + \sum_{k \neq j} x_k \beta_k^{(r_j)}\right) / \left(1 + \sum_{r_j=1}^{R_j-1} \exp\left(\alpha_{r_j} + \sum_{k \neq j} x_k \beta_k^{(r_j)}\right)\right) \quad (2)$$

# Generalized Free Model

## *Step 1: Clustering of manifest variables (2)*

Using of marginal test of likelihood ratio to calculate the intensity of link between  $X_j$  and  $X_k$ :

$$\Lambda(X_j/X_k) = -2 \left| l(\hat{\beta}_{(k)}, X_j) - l(\hat{\beta}, X_j) \right| \quad (3)$$

where  $\hat{\beta}$  is a vector of the parameters associated to the full model and  $\hat{\beta}_{(k)}$  is a vector of the parameters without  $X_k$

$$p\text{-value}(X_j/X_k) = \Pr[\chi_{v_k}^2 > \Lambda_{obs}(X_j/X_k)] \quad (4)$$

Calculation of the dissimilarity between  $X_j$  and  $X_k$ :

$$d(X_j, X_k) = \min[p\text{-value}(X_j/X_k) ; p\text{-value}(X_k/X_j)] \quad (5)$$

# Generalized Free Model

## *Step 1 : Clustering of manifest variables (3)*

We obtain a symmetric matrix of dissimilarities  $(p,p)$

The number of clusters  $G$  is chosen with the **Ward's hierarchical ascending clustering** on this matrix :

- (i) The simplest rule cuts the clustering tree where there is a big loss of inter-cluster inertia
- (ii) The improvement of this criterion :

$$R_W^2(G) > 1 - \frac{\alpha^2(p-G)}{T} \quad (6)$$

where  $R_W^2(G)$  is the proportion of within-cluster inertia and  $T$  is the total inertia

$\Rightarrow$  The final result consists in  $G$  clusters, containing  $(q_1, \dots, q_G)$  manifest variables

# Generalized Free Model

## *Step 2: Building the latent variables (1)*

Each cluster contains  $q_g$  variables which can be measured on different scales, with  $\sum_{g=1}^G q_g = p$  where it can have missing data

For each cluster, the GPCA is applied with aid of the **Generalized NIPALS algorithm**. This technique deals with too missing data according to the same principle as the NIPALS algorithm

Only the first principal component is used as in RFPC

⇒ The final result consists in  $G$  latent variables,  $(Z_1, \dots, Z_G)$

*Particular case*: if a block of variables are only on ordinal scale, without missing data we obtain the first principal component of the Multiple Correspondence Analysis

# Generalized Free Model

## *Step 2: Building the latent variables (2)*

Methods to apply in accordance with the type of variables and missing data on not to obtain a latent variable

	<b>Numeric</b>	<b>Binary or ordinal</b>	<b>Nominal</b>	<b>Count</b>	<b>Mixed</b>
Not missing	PCA	MCA	MCA	Generalized NIPALS	Generalized NIPALS
Missing	NIPALS	Categorical NIPALS	Categorical NIPALS	Generalized NIPALS	Generalized NIPALS

# Generalized Free Model

## *Step 3: Searching for links between latent variables*

All the  $G$  latent variables are on a numeric scale

The partial correlation between each couple  $(Z_s, Z_q)$  giving the  $G-2$  other  $Z_g$ :  $\rho(Z_s, Z_q / \forall Z_g, g \neq s, q)$  is calculated

The associated statistic of test follows a Student's distribution with  $n-G-1$  deg. of freedom under  $H_0$  (non-correlation between  $Z_s$  and  $Z_q$ )

The link between two latent variables is considered significant if the  $p$ -value is lower than a fixed threshold (for instance : 0.01)

This procedure leads to  $K \in [0, G(G-1)/2]$  links

$\Rightarrow$  The final result is a non-directed graph

# Generalized Free Model

## *Step 4: Orientation of links between latent variables*

It is usually preferable to use an expert decision to avoid a non efficient model

However, different statistic solutions can be proposed

If there are  $K$  significant links, then the number of different models is equal to  $2^K$

Then, each model can be judged by a score, (GoF, global  $R^2$ , ...), and an expert can choose among the ten highest scores, for instance (these models must not have cycle in the directed graph)

⇒ The final result is a directed graph



# Generalized Free Model

## *Step 5: Estimation of the generalized free model*

The free model is estimated through the **RFGPC** approach (Regression on the First Generalized Principal Components) [Derquenne, 2005] or by the **PML** approach (Partial Maximum Likelihood) [Derquenne, 2005]

The **internal model** is estimated by means of multiple linear regression between latent variables

The **external model** is estimated by means regression between each latent variable and its own manifest variables

⇒ The final result is the generalized free model



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# Bayesian Networks approach

## Bayesian Networks (BN) = graphical form

- Nodes = model's variables
- Edges = direct probability relations
- Powerful tools for non-supervised learning [Jensen, 2001]
- Used to find links between observed variables in order to create a robust outer model

## Why to use BN for customer satisfaction surveys ?

- Handled using questions with nominal or ordinal scales

# Bayesian Networks approach

## Overall principle (1) :

- Using structure learning heuristics based on network scores by **Minimum Description Length (MDL)**
- Search based on a global characterization of the data and on the exploitation of the properties of equivalent Bayesian networks with **SopLEQ** algorithm [Jouffe, 2004]
- Building a **segmentation of variables** with aid of this graph is applied to obtain clusters of nodes (corresponding to clustering variables = **step 1 of GFM**)
- For each cluster, a **categorical latent variable** is built by clustering the individuals, as a new node (**step 2**) and the external model is created

# Bayesian Networks approach

## Overall principle (2) :

- Search relations between these new nodes with SopLEQ again on these new nodes (step 3)
- Build the structural model, the position of links (step 4) is directly given by SopLEQ (possibly to choose a different position of links among equivalent Bayesian networks)
- Estimation of « Free model » is given by making inference on full model (external and internal models) in selecting a target node among latent variables (step 5)

# Bayesian Networks approach

## Comparison of the both approaches

<i>Steps</i>	<i>Bayesian Networks</i>	<i>Generalized Free Models</i>
Searching links between manifest variables	SopLEQ Algorithm	Building dissimilarities matrix ( $\rho$ values) between each couple of variables by Generalized Linear Models
Clustering of variables	Segmentation of variables	Ward's distance on the dissimilarities matrix
Building latent variables	Segmentation of data	1 <sup>st</sup> generalized principal component
Searching links between latent variables	SopLEQ	Partial correlation test
Position of links between latent variables	Algorithm	By expert ou by selection of the "best" statistical model
Estimation of the Generalized Free Model		RFGPC approach or PML approach



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# Application to marketing data

## Satisfaction and loyalty survey concerning electric central heating

**Context :** The marketing Department would like to evaluate the leverage of loyalty and satisfaction with respect to central heating. But, it doesn't exist a specific marketing conceptual model. However a satisfaction and loyalty survey has been conduct, but the questionnaire was not explicitly built to be used for a path modeling. Then this department asks to R&D to construct a first statistical model as an initial marketing model



# Application to marketing data

## Form of the questionnaire and results of survey :

Several parts :

- socio-economics questions (age, type of household, ...)
- specific questions (satisfaction, expectations, loyalty, ...)

Results : The answers are distributed on semantic scales

- 4 binary variables
- 5 nominal variables
- 19 ordinal variables
  
- 6899 questionnaires
- Some contain missing data

# Application to marketing data

## Results on the Generalized Free Model (1)

### *Step 1* : Clustering of manifest variables (8 clusters)

Expectation #1 (EXPEC\_1:  $q_1 = 5$ )

Expectation #2 (EXPEC\_2:  $q_2 = 4$ )

Socio-demographics (SOC\_DEM:  $q_3 = 4$ )

Housing (HOUSING:  $q_4 = 4$ )

Customer's contract (CONTRA:  $q_5 = 2$ )

Loyalty (LOYALTY:  $q_6 = 2$ )

Satisfaction/recommendation (SAT-REC:  $q_7 = 3$ )

Perceived value (PER-VAL:  $q_8 = 7$ )

### *Step 2* : Building the 8 latent variables :

EXPEC\_1 ; EXPEC\_2 ; SOC\_DEM ; HOUSING ; CONTRA ; LOYALTY ; SAT-REC ; PER-VAL

# Application to marketing data

## Results on the Generalized Free Model (2)

*Step 3*: Searching for links between latent variables

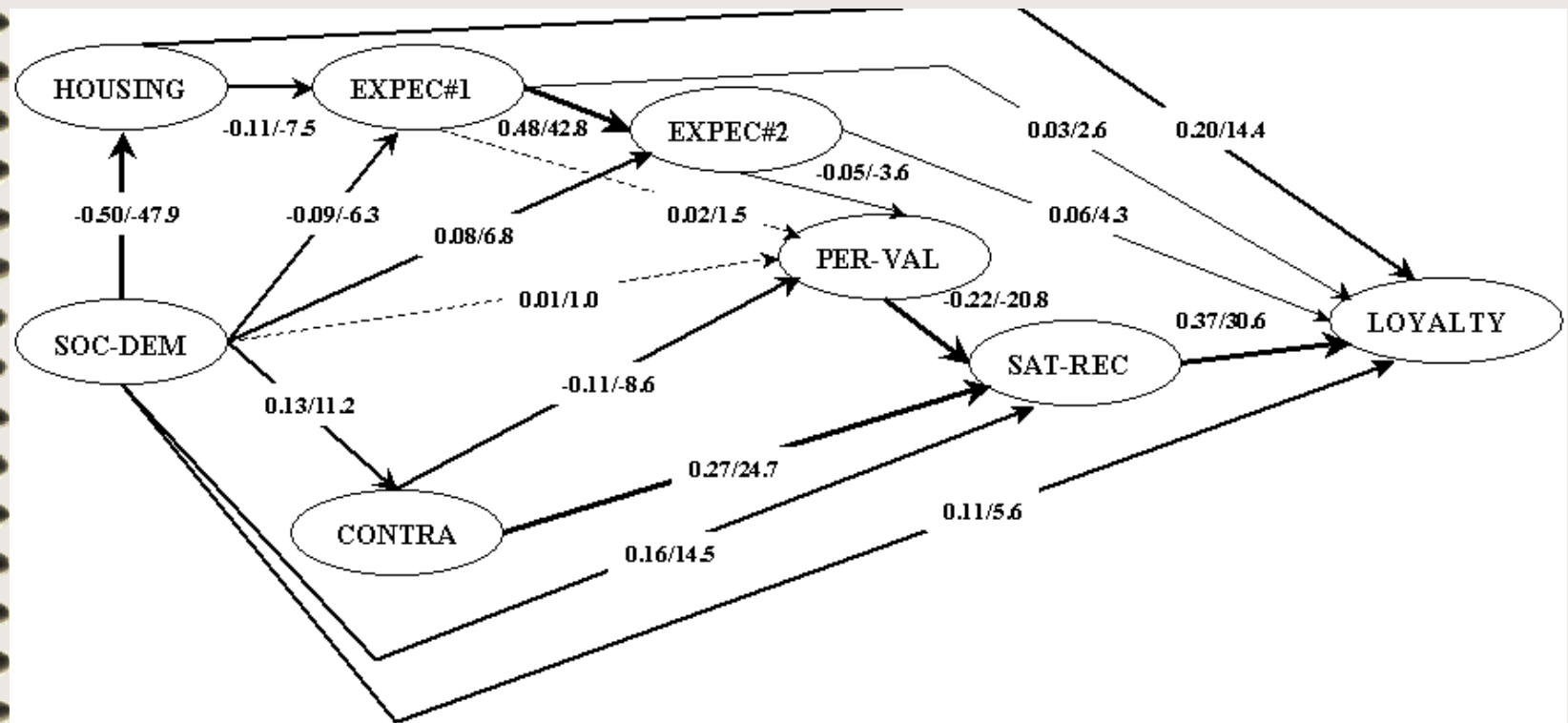
*Step 4*: Orientation of links between latent variables

Couple of latent variables	Partial correlation	$p$ -value	Position of links
(EXPEC#1,EXPEC#2)	+0.6311	< 0.0001	EXPEC#1→EXPEC#2
(LOYALTY,SAT-REC)	+0.5024	< 0.0001	SAT-REC→LOYALTY
(SOC-DEM,HOUSING)	+0.4004	< 0.0001	SOC-DEM→HOUSING
(SAT-REC,PER-VAL)	-0.2560	< 0.0001	PER-VAL→SAT-REC
(SAT-REC,HOUSING)	+0.1555	< 0.0001	HOUSING→SAT-REC
(EXPEC#1,SOC-DEM)	-0.1449	< 0.0001	SOC-DEM→EXPEC#1
(LOYALTY,HOUSING)	-0.1290	< 0.0001	HOUSING→LOYALTY
(SOC-DEM,LOYALTY)	+0.1217	< 0.0001	SOC-DEM→LOYALTY
(CONTRA,SAT-REC)	+0.1022	< 0.0001	CONTRA→SAT-REC
(EXPEC#2,SOC-DEM)	+0.1005	< 0.0001	SOC-DEM→EXPEC#2
(SOC-DEM,PER-VAL)	+0.0893	< 0.0001	SOC-DEM→OPINION
(SOC-DEM,SAT-REC)	+0.0828	< 0.0001	SOC-DEM→SAT-REC
(EXPEC#1,LOYALTY)	-0.0735	< 0.0001	EXPEC#1→LOYALTY
(EXPEC#1,HOUSING)	+0.0697	< 0.0001	EXPEC#1→HOU-EDF
(EXPEC#2,OPINION)	+0.0649	< 0.0001	EXPEC#2→OPINION
(EXPEC#2,LOYALTY)	+0.0471	0.0001	EXPEC#2→LOYALTY
(CONTRA,SOC-DEM)	+0.0469	0.0001	SOC-DEM→CONTRA
(CONTRA,PER-VAL)	-0.0394	0.0011	CONTRA→PER-VAL
(EXPEC#1,PER-VAL)	-0.0353	0.0034	EXPEC#1→PER-VAL

# Application to marketing data

## Results on the Generalized Free Model (3)

*Step 5:* Estimation of the generalized free model by RFGPC



# Application to marketing data

## Results on the Generalized Free Model (4)

### Evaluation of the Structural Model

Latent Variables	R2	Adjusted R2	Communalities	Redundancies
EXPEC#1	0.0105	0.0102	0.5915	0.0062
EXPEC#2	0.2266	0.2264	0.5158	0.1169
CONTRA	0.0177	0.0176	0.7005	0.0124
LOYALTY	0.1506	0.1500	0.5632	0.0848
HOUSING	0.2495	0.2494	0.4958	0.1237
PER-VAL	0.0132	0.0126	0.6141	0.0081
SAT-REC	0.1773	0.1770	0.5010	0.0888
SOC_DEM	.	.	0.4820	.

GoF = 0.2621

# Application to marketing data

## Results on the Bayesian Network

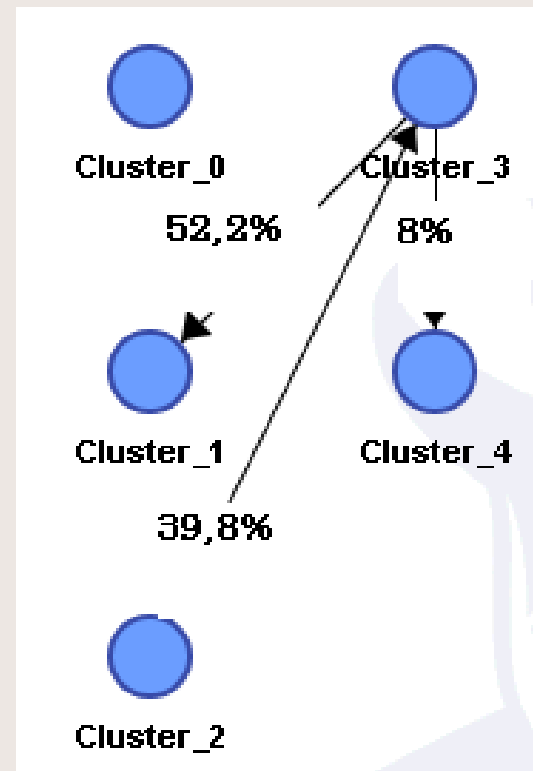
Cluster\_0 = {EXPEC#1 ; EXPEC#2}

Cluster\_1 = {HOUSING ; SOC\_DEM}

Cluster\_2 = {PER-VAL}

Cluster\_3 = {LOYALTY ; SAT-REC ; (-X<sub>j</sub>) }

Cluster\_4 = {CONTRA ; (+ X<sub>j</sub>)}





# Outline

- 1 Issues and motivations
- 2 How to build a free model ?
- 3 Generalized Free Model approach
- 4 Bayesian Networks approach
- 5 Application to marketing data
- 6 **Concluding remarks, applications and perspectives**

# Concluding remarks, applications and perspectives

The proposed method allows to build a **free model with variables measured on mixed scales** and can easily be applied in marketing, chemiometrics, policy, human capital, ...

Other methods for creating models exist: **stepwise structural equation modelling** [Hui, 1982] or **bayesian networks** [Jensen, 2001]

*Remaining works :*

- To compare with the other methods
- Future researches concern the improvement of the clustering step



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